Performance Analysis Of Harmonic Elimination In Cascaded H-Bridge Multilevel Inverters Using Constrained PSO Algorithm

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ABSTRACT: This paper focus on elimination of harmonics in a Cascaded H-bridge Multilevel Inverter by applying constrained Particle Swarm Optimization (PSO) algorithm. The expression for output voltage with fundamental equal to desired voltage and lower order harmonics equal to zero is considered as objective function. The proposed algorithm is utilized to obtain the optimized switching angles that reduce the effect of specific higher order non triplen harmonics from the output voltage of the multilevel inverter. In this paper, three phase 5 level Cascaded H-Bridge Inverter is considered to compare and analyze the level of harmonics with variation in modulation index. The simulation studies are extended to further 7-level, 9-level and 11-level to validate if same code can produce better results for obtaining more switching angles. The Simulation results indicate that constrained PSO based SHE effectively outperforms SPWM in terms of minimizing THD and provides high-quality output voltage waveform.

KEYWORDS: Harmonics, Multilevel inverter, Particle Swarming Optimization, Selective Harmonic Elimination, Sinusoidal Pulse Width Modulation.

1. INTRODUCTION

So far, Multilevel Inverters (MLIs) have attracted much attention for high power and high voltage applications [1, 13-15]. Among various topologies of multilevel converters, the cascaded H-bridge multilevel inverters are most suitable due to high degree of modularity, possibility of connecting directly to medium voltage, high power quality, high availability, and simplicity of control. The cascaded H-bridge multilevel inverter consists of cascade connection of an ‘m’ number of single-phase full-bridge inverters to generate a (2m + 1) number of levels. The principle of operation of cascaded H-bridge multilevel inverter is
usually based on synthesizing the desired output voltage waveform from several steps of
temperature obtained from separate dc voltage sources [2, 5, and 7].

Conventionally, the Sinusoidal Pulse Width Modulation [4, 8, and 16] and Space Vector
Modulation [3, 9, and 12] techniques are used to eliminate & minimize the low order voltage
harmonics at output & maintain the fundamental voltage at the desired value, but still
complexities are there such as time consuming as they are iterative and need heavy
computation burden and slow convergence. But for specific higher order harmonics, selective
harmonic elimination or programmed PWM methods are used.

Selective Harmonic Elimination (SHE) refers to the selection of switching angles such that
specific higher order harmonics such as the 5th, 7th, 11th, and 13th are eliminated from the
output voltage of the inverter. It requires solving nonlinear transcendental equations which
contain trigonometric terms, which can be solved by iterative techniques such as the Newton–
Raphson method. However, it becomes difficult to solve large number of equations, if good
initial guess is not available. The theory of resultants of polynomials [1] and the theory of
symmetric polynomials have also been suggested where the transcendental equations are
converted into the polynomial equations. However, the order of the polynomials become very
high if multilevel inverter employs several H-bridges connected in series, thereby making the
computations of the solutions of these polynomials very complex.

Genetic Algorithm (GA) based optimization technique was also proposed for computing
switching angles of multilevel inverter [10, 11]. But, the quality of solution deteriorates with
increase in the level of inverter as implementation of this method requires proper selection of
certain parameters such as population size, mutation rate, etc. Therefore, we have utilized
SHE technique using the application of PSO to obtain minimum harmonic distortion and
desired fundamental voltage. In this paper, comparative analysis of harmonic elimination in
cascaded H-bridge multilevel inverter with variation in modulation index has been done.

2. CASCADED MULTILEVEL INVERTER

The cascaded H-bridge multilevel inverter consists of a series of (m-1)/2 single-phase H-
bridge inverter units, with each unit connected to separate DC source. Each H-bridge has four
switches capable of generating three different voltage outputs, +Vdc, 0, and –Vdc. The output
phase voltage V_an of m-level CH-MLI is the sum of individual H-bridge inverter unit outputs.
\[ V_{an} = V_1 + V_2 + V_3 + \ldots \ldots + V_{(m-1)/2} \] (1)

where \( V_1, V_2, V_3, \ldots, V_{(m-1)/2} \) are the output voltages of each H-bridge inverter units.
Figure 1a) Basic configuration of Single Phase Cascaded H-Bridge Multilevel Inverter, b) Output waveform of Single Phase Cascaded H-Bridge Multilevel Inverter

The instantaneous output voltage of cascaded H-bridge m-level inverter can be represented by Fourier series as

\[ V(t) = \sum_{n=1}^{\infty} (a_n \sin n\alpha_n + b_n \cos n\alpha_n) \]

(2)

where

\[ a_n = \frac{4V_{dc}}{n\pi} \sum_{k=1}^{(m-1)/2} \cos n\alpha_k \]

and \( b_n = 0 \), due to quarter wave symmetry.

and all switching angles \( \alpha_1, \alpha_2, \alpha_3 \ldots \ldots \ldots \ldots, \alpha_{(m-1)/2} \) must satisfy the condition

\[ 0 < \alpha_1 < \alpha_2 < \alpha_3 < \ldots \ldots \ldots < \alpha_{(m-1)/2} < \frac{\pi}{2} \]

As quarter symmetry of the waveform results in elimination of the even harmonics and the dc constant, only odd harmonics are required to be considered. Hence, only odd harmonics are required to be considered. And in balanced three phase systems all triplen harmonics are zero.

\[ V(t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi} \left( \cos n\alpha_1 + \cos n\alpha_2 + \cos n\alpha_3 \ldots \ldots + \cos n\alpha_{(m-1)/2} \right) \sin n\alpha_n \]

(3)

To obtain minimum harmonics, fundamental component of output voltage is should be equal to desired voltage and lower order odd harmonics should be zero. So, the above equation reduces to following equations to satisfy the required constraints.

\[ \begin{align*}
\cos \alpha_1 + \cos \alpha_2 + \cos \alpha_3 \ldots \ldots + \cos \alpha_{(m-1)/2} &= \frac{(m-1)M}{2} \\
\cos 3\alpha_1 + \cos 3\alpha_2 + \cos 3\alpha_3 \ldots \ldots + \cos 3\alpha_{(m-1)/2} &= 0 \\
\cos 5\alpha_1 + \cos 5\alpha_2 + \cos 5\alpha_3 \ldots \ldots + \cos 5\alpha_{(m-1)/2} &= 0 \\
\cos 7\alpha_1 + \cos 7\alpha_2 + \cos 7\alpha_3 \ldots \ldots + \cos 7\alpha_{(m-1)/2} &= 0 ; \text{and so on.}
\end{align*} \]

(4)
The solution of these nonlinear transcendental equations gives the value of switching angles at which individual inverter units should be turned on so as to obtain low THD. Conventionally, these equations are solved by iterative techniques such as the Newton–Raphson method. However, it becomes difficult to solve large number of equations in cases involving large number of switching angles.

### 3. PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) is a population based stochastic optimization algorithm developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling. PSO refers to a relatively new family of algorithms that may be used to find optimal solutions to numerical and qualitative problems. PSO algorithm employs a population of randomly generated particles (which are basically potential solution to the optimization problem) that moves stochastically in the search space according to simple mathematical formulae. Each particle keeps track of its movement, which is associated with the best solution (Pbest) found so far. This best solution is guided toward the other best known solutions in the search-space, which are updated as better solutions are found by other particles. This solution is commonly called Gbest.

The basic concept behind the PSO technique is to change the velocity of each particle towards its ‘Pbest’ and ‘Gbest’ positions at each time step. This algorithm is easily implemented in most programming languages and has proven to be both very fast and effective when applied to a diverse set of optimization problem. The position and velocity of the \( i \)th particle of a \( d \)-dimensional search space can be represented as

\[
X_i = [x_{i1} \ x_{i2} \ x_{i3} \ \ldots \ \ldots \ \ldots \ x_{id}]
\]

\[
V_i = [v_{i1} \ v_{i2} \ v_{i3} \ \ldots \ \ldots \ \ldots \ v_{id}]
\]

The elements of \( X_i \) are the solutions of the harmonic minimization problem.

The best previous position of a particle is evaluated using objective function and then recorded and represented as:

\[
P_{best_i} = [p_{i1} \ p_{i2} \ p_{i3} \ \ldots \ \ldots \ \ldots \ p_{id}]
\]

Here, Pbest is the local best solution. Now, let \( g_0 \) particle is best among all particles in the group, it is represented as Gbest, i.e. global best position.
\[ G_{best} = P_{bestg} = [P_{g1} \ P_{g2} \ldots \ldots \ldots \ldots P_{gd}] \]

In PSO, optimal solution depends upon Gbest and Pbest. The position of particle is updated every time using the current velocity and the distance from Pbest and Gbest. The velocity and position of each particle can be updated using the following equations:

\[
v_{id}^{k+1} = w \cdot v_{id}^{k} + c_1 \cdot r_1 \cdot (P_{bestid} - x_{id}^{k}) + c_2 \cdot r_2 \cdot (G_{bestgd} - x_{id}^{k})
\]

\[
x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1}
\]

where \( w \) is the inertia weight parameter, which controls the global and local exploration capabilities of the particle. And \( c_1 \) and \( c_2 \) are the constraints of cogitative and social task respectively. Meanwhile, \( r_1 \) and \( r_2 \) are random numbers between 0 and 1.

4. PROPOSED ALGORITHM

The objective function for minimization of THD in multilevel inverters using this constrained PSO algorithm is chosen to obtain the switching angles using the function:

\[
Objective.\ Function = \left(V_1 - \frac{(m-1) \cdot M}{2}\right) + \sum_{k=5,7,11}^{49} \frac{V_k}{k}
\]

where \( V_1 = \cos \alpha_1 + \cos \alpha_2 + \cos \alpha_3 + \ldots + \cos \frac{(m-1)}{2} \alpha \)

\[
V_k = \cos k\alpha_1 + \cos k\alpha_2 + \cos k\alpha_3 + \ldots + \cos k\frac{(m-1)}{2} \alpha
\]

Here, \( M \) is the modulation index. And \( \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{(m-1)/2} \) are the switching angles of \( m \)-level inverter with constraints:

\[
0 < \alpha_1 < \alpha_2 < \alpha_3 < \ldots < \frac{(m-1)}{2} \alpha < \frac{\pi}{2}
\]

For three phase 5-level CH-MLI, objective function reduces to:

\[
Objective.\ Function.\ 5\text{level} = (V_1 - 2 \cdot M) + \sum_{k=5,7,11}^{49} \frac{V_k}{k}
\]

where \( V_1 = \cos \alpha_1 + \cos \alpha_2 \)
\[ V_k = \cos k\alpha_1 + \cos k\alpha_2 \]  \hspace{1cm} (8)

Here, on the basis of observation with different constraints, the constraints are modified as:

\[
\begin{align*}
0 &< \alpha_1 < 45 \\
22.5 &< \alpha_2 < 90 \\
\alpha_1 &< \alpha_2 \\
\alpha_1 - \alpha_2 &< 45
\end{align*}
\]

The proposed PSO algorithm is given as following steps:
Step 1. Input the data: Modulation Index \( M \), Population size, Maximum iterations, Maximum runs, Inertia Weight, and Acceleration Factors.
Step 2. Create the random initial population of switching angles by considering their upper and lower bound.
Step 3. Initialize the velocity, Pbest, Gbest, iteration count for computing switching angles.
Step 4. Update the iteration count, velocity and position for moving of particles in search space.
Step 5. Evaluate the fitness using \( F(\alpha) \): Objective function satisfying the nonlinear constraints.
Step 6. Update the values of Pbest and Gbest.
Step 7. Check if all constraints are satisfied and global best value is achieved then go for next step otherwise repeat the Step 3 to Step 6 for best solution.
Step 8. Select the best solution of fitness value.

In this process the maximum iterations are 1000, maximum runs are 10 and the population size is 100 and using the decreasing inertia function with initial weight of 0.9 and final weight of 0.4. This proposed constrained PSO algorithm is used to obtain the switching angles, which are further used to generate pulses for firing the switches of the multilevel inverter. The best values of switching angles are obtained by updating the velocity, Pbest and Gbest values so that harmonic content in the output voltage is minimum.

5. SIMULATION AND RESULTS

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A three phase five level Cascaded H-bridge Inverter with nonlinear RL load is simulated in MATLAB/Simulink for study of harmonic content in output voltage. PSO based modulation technique is designed to generate switching pulses for multilevel inverter, and compared with SPWM. The m-file is utilized to compose the algorithm codes for obtaining the switching angles for multilevel inverter. The obtained switching angles are converted into switching pulses for the switches of H-bridge. The Line voltages and Fourier transform (FFT) analysis for both the cases are shown in Fig. It is evident from the harmonic spectrum that the lower order harmonics reduces significantly.

Figure 2 a) Simulink model of three phase five-level cascaded H-bridge inverter employing PSO technique, b) Sub-circuit of three phase five level cascaded H-bridge inverter

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The THD value in three phase five-level cascaded H-bridge inverter is found to be 11.53% using SPWM. The five-level cascaded H-bridge inverter uses two H bridges and the switching angles obtained for generating switching pulse using PSO are $\alpha_1 = 9.4248$ and $\alpha_2 = 34.5575$ in degree. The THD using PSO is found to be 10.39%. The THD further reduces at increased values of modulation indices. The principle goal of this work is to wipe out the selective low order harmonics like fifth, seventh order harmonics and so on. The THD acquired from PSO is low contrasted with SPWM. Also, the fundamental voltage obtained using PSO technique $V_1 = 4374\, \text{V}$ is close to 4.4KV and better than that obtained using SPWM $V_1 = 3279\, \text{V}$. 

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Comparative analysis of harmonic level among SPWM and PSO technique is done with variation in modulation index from 0.85 to 1.05. The result of optimized switching angles obtained from proposed PSO technique that reduces the selected low order harmonics and minimizes THD% from three phase symmetric five-level cascaded H-bridge inverter with two separate DC sources is presented in Table 1 for different modulation index. And the comparison of THD obtained in 5-level CH-MLI using both the techniques is presented in Table 2.

Table 1 Switching angles for 5-level cascaded H-bridge inverter obtained using PSO

<table>
<thead>
<tr>
<th>Modulation Index (m)</th>
<th>$\alpha_1$ (degrees)</th>
<th>$\alpha_2$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>28.2479</td>
<td>59.6848</td>
</tr>
<tr>
<td>0.9</td>
<td>22.0036</td>
<td>59.6919</td>
</tr>
<tr>
<td>0.95</td>
<td>9.4248</td>
<td>28.2743</td>
</tr>
<tr>
<td>1.0</td>
<td>9.4248</td>
<td>34.5575</td>
</tr>
<tr>
<td>1.05</td>
<td>9.4399</td>
<td>21.9893</td>
</tr>
</tbody>
</table>

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Table 2 Comparison of THD at different Modulation Indices for 5-level cascaded H-bridge inverter

<table>
<thead>
<tr>
<th>Modulation Index (m)</th>
<th>THD (%) using SPWM</th>
<th>THD (%) using PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>16.11</td>
<td>15.59</td>
</tr>
<tr>
<td>0.9</td>
<td>15.89</td>
<td>12.01</td>
</tr>
<tr>
<td>0.95</td>
<td>12.99</td>
<td>10.54</td>
</tr>
<tr>
<td>1.0</td>
<td>11.53</td>
<td>10.39</td>
</tr>
<tr>
<td>1.05</td>
<td>11.19</td>
<td>9.79</td>
</tr>
</tbody>
</table>

Figure 5 THD Vs Modulation Index for 5-level cascaded H-bridge inverter using SPWM and PSO

It is observed that as the modulation index increases from 0.85 to 1.05, THD of output voltage decreases for both PSO and SPWM. However, the harmonic content and specifically lower order harmonics are less as compared to SPWM using constrained PSO algorithm. Hence, the quality of output voltage is better in PSO as compared to SPWM. The study is further extended to 7-level, 9-level and 11-level inverter and the results are presented in Table 3, 4 and 5.

Table 3 Comparison of THD at different Modulation Indices for 7-level cascaded H-bridge inverter

<table>
<thead>
<tr>
<th>Modulation Index (m)</th>
<th>THD (%) using SPWM</th>
<th>THD (%) using PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>10.20</td>
<td>8.63</td>
</tr>
<tr>
<td>0.9</td>
<td>9.30</td>
<td>8.37</td>
</tr>
<tr>
<td>0.95</td>
<td>9.16</td>
<td>7.23</td>
</tr>
<tr>
<td>1.0</td>
<td>8.13</td>
<td>6.37</td>
</tr>
<tr>
<td>1.05</td>
<td>8.04</td>
<td>6.11</td>
</tr>
</tbody>
</table>

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Table 4 Comparison of THD at different Modulation Indices for 9-level cascaded H-bridge inverter

<table>
<thead>
<tr>
<th>Modulation Index (m)</th>
<th>THD (%) using SPWM</th>
<th>THD (%) using PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>7.51</td>
<td>6.43</td>
</tr>
<tr>
<td>0.9</td>
<td>7.02</td>
<td>6.30</td>
</tr>
<tr>
<td>0.95</td>
<td>7.33</td>
<td>6.06</td>
</tr>
<tr>
<td>1.0</td>
<td>6.22</td>
<td>5.80</td>
</tr>
<tr>
<td>1.05</td>
<td>5.98</td>
<td>5.70</td>
</tr>
</tbody>
</table>

Table 5 Comparison of THD at different Modulation Indices for 11-level cascaded H-bridge inverter

<table>
<thead>
<tr>
<th>Modulation Index (m)</th>
<th>THD (%) using SPWM</th>
<th>THD (%) using PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>5.92</td>
<td>5.655</td>
</tr>
<tr>
<td>0.9</td>
<td>5.18</td>
<td>5.24</td>
</tr>
<tr>
<td>0.95</td>
<td>5.63</td>
<td>4.82</td>
</tr>
<tr>
<td>1.0</td>
<td>4.71</td>
<td>4.46</td>
</tr>
<tr>
<td>1.05</td>
<td>5.14</td>
<td>4.23</td>
</tr>
</tbody>
</table>

Figure 6 Graph showing THD in different MLI using SPWM and PSO technique

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6. CONCLUSION

In this paper, constrained PSO code is developed for obtaining the switching angles so that THD produced in the output voltage waveform is reduced. The Simulink model of 5-level cascaded H-bridge inverter is developed in MATLAB/Simulink environment to validate the results obtained from constrained PSO code. The results obtained from PSO based 5-level cascaded inverter are compared with those obtained from SPWM based 5-level cascaded inverter and found to be better. FFT analysis for SPWM and PSO based 5-level cascaded inverter has been carried out for different modulation indices. The simulation results clearly indicate that switching angles obtained from PSO code produce much better waveform than SPWM. The simulation has taken about 10 to 12 seconds only. The output voltage has much lower THD using PSO than SPWM. The proposed method shows better results when extended to obtaining more switching angles for 7-level, 9-level and 11-level, and produce low THD voltage output. Thus, constrained PSO is suitable for harmonic elimination in MLIs.

REFERENCES


